Problem 89

The purpose of this problem is to show the entire concept of dimensional consistency can be summarized by the old saying "You can't add apples and oranges." If you have studied power series expansions in a calculus course, you know the standard mathematical functions such as trigonometric functions, logarithms, and exponential functions can be expressed as infinite sums of the form $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$, where the a_n are dimensionless constants for all $n = 0, 1, 2, \cdots$, and x is the argument of the function. (If you have not studied power series in calculus yet, just trust us.) Use this fact to explain why the requirement that all terms in an equation have the same dimensions is sufficient as a definition of dimensional consistency. That is, it actually implies the arguments of standard mathematical functions must be dimensionless, so it is not really necessary to make this latter condition a separate requirement of the definition of dimensional consistency as we have done in this section.

Solution

Requiring all terms in an equation to have the same dimensions makes it so that a_0, $a_1 x$, $a_2 x^2$, ... all have the same dimensions. But this means that x, the function argument, has to be dimensionless because otherwise raising x to different powers would result in terms with different dimensions.